

# Constraints from the muon $g - 2$ on the parameter space of the NMSSM

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**Florian Domingo and Ulrich Ellwanger**

*Laboratoire de Physique Théorique,\**

*Université de Paris XI, F-91405 Orsay Cedex, France*

*E-mail: [domingo@th.u-psud.fr](mailto:domingo@th.u-psud.fr), [ellwanger@th.u-psud.fr](mailto:ellwanger@th.u-psud.fr)*

**ABSTRACT:** We generalize the computation of supersymmetric contributions to the muon anomalous magnetic moment  $(g - 2)_\mu$  to the NMSSM. In the presence of a light CP-odd Higgs scalar, these can differ considerably from the MSSM. We discuss the amount of these contributions in regions of the parameter space of the general NMSSM compatible with constraints from B physics. In the mSUGRA-like cNMSSM, constraints from  $(g - 2)_\mu$  prefer regions in parameter space corresponding to a low SUSY breaking scale.

**KEYWORDS:** Supersymmetry Phenomenology, Beyond Standard Model, Supersymmetric Standard Model, Electromagnetic Processes and Properties.

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\*Unité mixte de Recherche — CNRS — UMR 8627

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**1. Introduction**

The result of the measurement of the anomalous magnetic moment of the muon  $a_\mu = (g_\mu - 2)/2$  by the E821 experiment at the Brookhaven National Laboratory (BNL) [1] can be considered as a possible hint for physics beyond the Standard Model (SM). The determination of the SM contributions to  $a_\mu$  requires – amongst others – to compute the leading order hadronic contribution  $a_\mu^{\text{HLO}}$ , which is the main source of the present SM uncertainty. Recent calculations of  $a_\mu^{\text{HLO}}$  via hadronic  $e^+e^-$  data [2–6] are in good agreement, and lead to  $\sim 3$  standard deviations between the experimental value for  $a_\mu$  and the SM prediction.

Alternatively,  $a_\mu^{\text{HLO}}$  can be determined from hadronic  $\tau$ -decays [7, 8], leading to an agreement within one standard deviation between the experiment and the SM prediction for  $a_\mu$ . However, a comparison of the  $\tau$  branching fractions into pions with the corresponding  $e^+e^-$  spectral functions reveals a discrepancy of  $\sim 4.5$  standard deviations [6] and requires assumptions on the pion form factor, isospin violating effects and vector meson mixings. Since  $e^+e^-$  data is more directly related to  $a_\mu^{\text{HLO}}$ , it is advocated to use the hadronic  $e^+e^-$  data only for a computation of  $a_\mu^{\text{HLO}}$  [5].

Additional contributions to  $a_\mu$  appear within supersymmetric extensions of the SM (see [9] for a recent review). One-loop [10–13] and two-loop [14–22] contributions have been evaluated within the Minimal Supersymmetric Extension of the Standard Model (MSSM). The conclusion is that the MSSM is able to explain the 3 standard deviations between the experiment and the SM, provided that the masses of the electroweakly interacting supersymmetric particles are not far above the electroweak scale [12, 13, 19, 20, 9]. On the other hand, the measured value for  $a_\mu$  provides constraints on the parameter space of the MSSM such as the positivity of the supersymmetric Higgs mass parameter  $\mu$ .

The purpose of the present paper is the study of constraints from the measured value for  $a_\mu$  on the parameter space of the Next-to-Minimal Supersymmetric Extension of the

Standard Model (NMSSM). First estimates of the contributions to  $a_\mu$  in the NMSSM have been performed in [23] (see also [24] for  $a_\mu$  in a similar  $U(1)'$  model), but subsequently we aim at an accuracy comparable to the one in the MSSM (limiting ourselves, however, to dominant logarithms at two-loop order). To this end, most of the corresponding formulas can be translated in a straightforward way from the MSSM to the NMSSM.

Nevertheless, the numerical results in the NMSSM can differ considerably from the ones in the MSSM: the NMSSM contains an additional gauge singlet superfield  $S$ , the vacuum expectation value of which generates the Higgs mass parameter  $\mu$  of the MSSM [25]. The CP-even and CP-odd components of  $S$  will generally mix with the neutral components of the two (MSSM) Higgs doublets  $H_u$  and  $H_d$ . Depending on the parameters of the NMSSM, the lightest neutral CP-odd Higgs scalar can be quite light [26] and lead to numerous new phenomena [27]. Such a light neutral Higgs scalar can also have an important impact on  $a_\mu$  [28]. In section 3 we will study possible effects of a light neutral CP-odd Higgs scalar on  $a_\mu$  in regions of the parameter space of the NMSSM, which satisfy present bounds from LEP [29] and  $B$ -physics [30].

In the remaining part of the introduction we briefly review the various SM contributions to  $a_\mu$  in order to clarify, which additional contribution would be desirable. The SM contributions to  $a_\mu$  can be split into pure QED contributions  $a_\mu^{\text{QED}}$  (known to four-loop order), leading order hadronic contributions  $a_\mu^{\text{HLO}}$ , next-to-leading order hadronic contributions  $a_\mu^{\text{HNLO}}$  (vacuum polarisation diagrams only) and  $a_\mu^{\text{LBL}}$  (light-by-light contributions only), and electroweak effects  $a_\mu^{\text{EW}}$ . The sum is to be compared with the experimental result [1]

$$a_\mu^{\text{EXP}} = 11\,659\,208.0(5.4)(3.3) \times 10^{-10} . \tag{1.1}$$

The QED contribution is [31]

$$a_\mu^{\text{QED}} = 11\,658\,471.8113(162) \times 10^{-10} \tag{1.2}$$

(taking into account estimated five-loop contributions [32]) with an error well below the experimental one. The remaining difference is

$$a_\mu^{\text{EXP}} - a_\mu^{\text{QED}} = (736.2 \pm 6.3) \times 10^{-10} . \tag{1.3}$$

Among the recent evaluations of the leading order hadronic contributions  $a_\mu^{\text{HLO}}$  [2–6] (including the most recent data from SND, CMD-2 and BaBar) we use — in order to remain conservative — the largest estimate from [2, 5], leading to the smallest deviation from the SM:

$$a_\mu^{\text{HLO}} = (692.1 \pm 5.6) \times 10^{-10} \tag{1.4}$$

For  $a_\mu^{\text{HNLO}}$  we use the same reference [2, 5]:

$$a_\mu^{\text{HNLO}} = (-10.03 \pm 0.22) \times 10^{-10} \tag{1.5}$$

The most recent determination of the hadronic light-by-light contribution [33] gives

$$a_\mu^{\text{LBL}} = (11.0 \pm 4.0) \times 10^{-10} . \tag{1.6}$$

Adding all errors quadratically, one obtains

$$a_\mu^{\text{EXP}} - a_\mu^{\text{QED+HAD}} = (43.1 \pm 9.3) \times 10^{-10} ; \quad (1.7)$$

the remaining discrepancy should be explained by electroweak effects and/or contributions beyond the SM as supersymmetry. Within the SM, the electroweak contributions are to two-loop order [34]

$$a_\mu^{\text{EW}} = (15.4 \pm 0.2) \times 10^{-10} , \quad (1.8)$$

which leads to the present discrepancy of about three standard deviations between the experiment and the SM:

$$a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = (27.7 \pm 9.3) \times 10^{-10} \quad (1.9)$$

In the next section 2 we will review the supersymmetric contributions to  $a_\mu$ , and specify their dependency on the parameters of the NMSSM. Contributions to  $a_\mu$  which are very similar in the NMSSM and the MSSM (without possibly light CP-odd Higgs scalars), and contributions which involve the possibly light pseudoscalar of the NMSSM, are treated in separate subsections. The formulas of section 2 will be made public in the form of a Fortran code on the NMSSMTools web page [35]. In section 3 we present numerical results for  $a_\mu$  in various regions of the parameter space of the NMSSM, amongst others regions with a light CP-odd neutral Higgs scalar in the spectrum, and in the cNMSSM (with universal soft terms at the GUT scale). At the end of section 3 we conclude with a short summary.

## 2. Supersymmetric contributions to $a_\mu$ in the NMSSM

### 2.1 Contributions without pseudoscalars

The one-loop contributions to  $a_\mu$  in supersymmetric models are known to consist in chargino/sneutrino or neutralino/smuon loops [10 – 13]. The corresponding expressions are the same in the MSSM and in the NMSSM, provided that the additional singlino state in the neutralino sector (which includes now five states) is taken into account. Using the formulas in [11], the chargino/sneutrino and neutralino/smuon contributions to  $a_\mu$  can be written as follows:

$$\delta a_\mu^{1L} \chi^\pm = \frac{m_\mu}{16\pi^2} \sum_k \left[ \frac{m_\mu}{12m_{\tilde{\nu}_\mu}^2} (|c_k^L|^2 + |c_k^R|^2) F_1^C(x_k) + \frac{2m_{\chi_k^\pm}}{3m_{\tilde{\nu}_\mu}^2} \text{Re}[c_k^L c_k^R] F_2^C(x_k) \right] \quad (2.1)$$

$$\delta a_\mu^{1L} \chi^0 = \frac{m_\mu}{16\pi^2} \sum_{i,m} \left[ -\frac{m_\mu}{12m_{\tilde{\mu}_m}^2} (|n_{im}^L|^2 + |n_{im}^R|^2) F_1^N(x_{im}) + \frac{m_{\chi_i^0}}{3m_{\tilde{\mu}_m}^2} \text{Re}[n_{im}^L n_{im}^R] F_2^N(x_{im}) \right] \quad (2.2)$$

Here  $c_k^R$ ,  $c_k^L$ ,  $n_{im}^R$  and  $n_{im}^L$  denote the couplings of the mass eigenstates of the charginos or neutralinos to the sneutrino or smuons and the muon, which depend on the corresponding

mixing matrices described below:

$$c_k^R = h_\mu U_{k2} \tag{2.3}$$

$$c_k^L = -g_2 V_{k1} \tag{2.4}$$

$$n_{im}^R = \sqrt{2} g_1 N_{i1} X_{m2}^{\tilde{\mu}} + h_\mu N_{i3} X_{m1}^{\tilde{\mu}} \tag{2.5}$$

$$n_{im}^L = \frac{1}{\sqrt{2}} (g_2 N_{i2} + g_1 N_{i1}) X_{m1}^{\tilde{\mu}*} - h_\mu N_{i3} X_{m2}^{\tilde{\mu}*} \tag{2.6}$$

The conventions for the mass matrices and the resulting mixing matrices are as in [36, 37, 11, 35]:

- The neutralino mass eigenstates  $\chi_i^0$ ,  $i = 1, \dots, 5$  in the NMSSM (ordered in increasing absolute mass) are given in terms of the interaction eigenstates  $\psi_j = (-i\lambda_1, -i\lambda_2, \psi_d^0, \psi_u^0, \psi_s)$  by  $\chi_i^0 = N_{ij} \psi_j^0$ .
- The chargino mass eigenstates  $\chi_k^\pm$ ,  $k = 1, 2$ , are related to the charged gaugino and higgsino interaction eigenstates  $\psi^+ = (-i\lambda^+, \psi_u^+)$  and  $\psi^- = (-i\lambda^-, \psi_d^-)$  through the rotation matrices  $U, V$ :  $\chi_k^+ = V_{kl} \psi_l^+$ ,  $\chi_k^- = U_{kl} \psi_l^-$ .
- The smuon mass eigenstates  $\tilde{\mu}_m$ ,  $m = 1, 2$ , with masses  $m_{\tilde{\mu}_m}$ , result from the interaction eigenstates  $\tilde{\mu}^I = (\tilde{\mu}_L, \tilde{\mu}_R)$  and the rotation  $\tilde{\mu}_m = X_{mn}^{\tilde{\mu}} \tilde{\mu}_n^I$ . In the following, this definition of the matrix  $X_{mn}^{\tilde{\mu}}$  will be extended to all sfermions  $\tilde{f}$ , in which case it will be denoted as  $X_{mn}^{\tilde{f}}$ .

The  $\mu$ -sneutrino mass is written as  $m_{\tilde{\nu}_\mu}$ .  $x_{im}$  and  $x_k$  denote the following mass ratios:  $x_{im} = m_{\chi_i^0}^2 / m_{\tilde{\mu}_m}^2$ ,  $x_k = m_{\chi_k^\pm}^2 / m_{\tilde{\nu}_\mu}^2$ . The functions  $F_{1,2}^{C,N}$  are given in the appendix.  $h_\mu$  is the muon Yukawa coupling:  $h_\mu = \frac{m_\mu}{v_d}$ , where  $v_d$  is the vacuum expectation value of the Higgs doublet which couples to down quarks and leptons.

Since contributions to  $a_\mu$  require a chirality flip, the formulae (2.1), (2.2) involve terms — apart from the prefactor  $m_\mu$  — which are proportional either to the muon mass (if the chirality flip occurs in the external legs) or to a chargino/neutralino mass (when it occurs in internal lines). In practice, the numerically dominant contributions usually originate from a chirality flip in the internal lines of the chargino/sneutrino contribution, i.e. the second term in (2.1). It is proportional to  $c_k^R$ , and hence to the Yukawa coupling  $h_\mu \sim m_\mu / \cos \beta \sim m_\mu \tan \beta$  (for  $\tan \beta \gg 1$ ).

An analysis of the chargino/sneutrino diagram reveals (see, e.g., [9]) that this contribution carries the sign of the Higgs mass parameter  $\mu$  ( $= \mu_{\text{eff}}$  in the NMSSM), hence a positive value for  $\mu$  is phenomenologically favoured. The contribution decreases with increasing sneutrino mass. For positive  $\mu$ , the chargino/sneutrino contribution can well resolve the discrepancy (1.9), provided the muon sneutrino is relatively light or  $\tan \beta$  is large; see, e.g., figures 1–3 in section 3. (For a light sneutrino *and* large  $\tan \beta$ , this contribution can even be larger than desired.)

It has been pointed out in [11, 12] that the neutralino/smuon contribution (2.2) could also explain the  $3\sigma$  discrepancy (1.9), if the bino is quite light and the smuon mass eigenstates are not too heavy. This bino/smuon contribution has the interesting property to be quite insensitive to  $\tan \beta$ .

In the NMSSM, a light neutralino could also be dominantly singlino-like. However, in this case its contribution to  $a_\mu$  is suppressed because of the weak couplings of the singlino to the MSSM sector.

In addition to these one-loop diagrams, two-loop contributions to  $a_\mu$  in the MSSM have been studied in [14–22]. First, we include large logarithms arising from QED corrections to one-loop diagrams as computed in [14]:

$$\delta a_\mu^{SUSY+QED} = \delta a_\mu^{1L\ SUSY} \left( 1 - \frac{4\alpha}{\pi} \ln \frac{M_{SUSY}}{m_\mu} \right) \quad (2.7)$$

This leads to a reduction by a few percents of the LO contributions.

Additional bosonic electroweak two-loop diagrams were computed in [20], which can be written as

$$\delta a_\mu^{2L\ Bos\ EW} = \frac{5 G_F m_\mu^2 \alpha}{24\sqrt{2} \pi^3} \left( c_L^{2L\ Bos} \ln \frac{m_\mu^2}{M_W^2} + c_0^{2L\ Bos} \right). \quad (2.8)$$

(Up to the more complicated Higgs sector of SUSY models, (2.8) contains two-loop electroweak SM contributions included in (1.8). We took care not to count the SM contribution twice.) Subsequently we confine ourselves to leading logarithmic contribution  $\sim c_L^{2L\ Bos}$ , which reads [20]

$$c_L^{2L\ Bos} = \frac{1}{30} \left[ 98 + 9c_L^h + 23 (1 - 4s_W^2)^2 \right]. \quad (2.9)$$

In the MSSM, the Higgs contribution  $c_L^h$  is of the form (see eq. (27) in [20])

$$c_L^h = \frac{\cos 2\beta M_Z^2}{\cos \beta} \left[ \frac{\cos \alpha \cos(\alpha + \beta)}{m_H^2} + \frac{\sin \alpha \sin(\alpha + \beta)}{m_h^2} \right], \quad (2.10)$$

where  $m_{h,H}$  and  $\alpha$  are the CP-even Higgs masses and the mixing angle in the MSSM. In order to generalize the Higgs contribution to the NMSSM, it is convenient to re-write eq. (2.10) in terms of elements of the (inverse) CP-even Higgs mass matrix  $M_S^{-2}$ , here in the basis  $(H_u, H_d)$ :

$$c_L^h = \cos 2\beta M_Z^2 \left[ (M_S^{-2})_{22} - \tan \beta (M_S^{-2})_{12} \right] \quad (2.11)$$

(Now it is straightforward to verify the sum rule  $c_L^h = 1$  [20] with the help of the tree level mass matrix  $M_S^2$ .)

Eq. (2.11) can be interpreted as the result of the evaluation of the Higgs-dependent two-loop diagrams, where Feynman rules in the interaction basis  $(H_u, H_d)$  have been used and mass insertions were treated perturbatively — this procedure reproduces the leading logarithms. Furthermore, in the interaction basis it is easy to identify the additional contributions from the singlet scalar of the NMSSM, which decouples from the muon as well as from gauge bosons. The only possible additional contributions arise from the coupling of the singlet to charged Higgs bosons (involving always at least one power of the Higgs singlet-doublet coupling  $\lambda$ ), and the simultaneous presence of a mass insertion transforming a doublet into a singlet (also of  $\mathcal{O}(\lambda)$ ). Since the coefficient of  $c_L^h$  in (2.8) is only  $\sim -2 \times 10^{-11}$ , we neglect subsequently effects of  $\mathcal{O}(\lambda^2/g^2)$  in  $c_L^h$ . Then eq. (2.11) is

equally valid in the NMSSM; all that remains to be done is to replace  $M_{\mathcal{G}}^2$  by the  $3 \times 3$  mass matrix of the CP-even sector of the NMSSM. Returning, for convenience, to mass eigenstates,  $c_L^h$  is then of the form

$$c_L^h = \cos 2\beta M_Z^2 \sum_{i=1}^3 \frac{S_{i2} (S_{i2} - \tan \beta S_{i1})}{m_{h_i}^2}. \quad (2.12)$$

Here, the conventions for the Higgs mixing matrices  $S_{ij}$  are as follows [35] (for convenience we discuss the complete Higgs sector here, which is useful for what follows below; note that  $H_u$  and  $H_d$  are exchanged w.r.t. the conventions in [36, 37]):

- The CP even Higgs eigenstate  $h_i$ ,  $i = 1, 2, 3$  (ordered in increasing mass) in the NMSSM are a mixture of the real parts of the neutral Higgs components  $S^I = (H_{uR}, H_{dR}, S_R)$ :  $h_i = S_{ij} S_j^I$ .
- The CP odd Higgs states  $a_i$ ,  $i = 1, 2$  in the NMSSM originate from the imaginary parts of the neutral Higgs components  $P^I = (H_{uI}, H_{dI}, S_I)$  (after omission of the Goldstone boson  $G^0 = -\sin \beta H_{uI} + \cos \beta H_{dI}$ ):  $a_i = P'_{i1} (\cos \beta H_{uI} + \sin \beta H_{dI}) + P'_{i2} S_I$ .
- The charged Higgs boson  $H^\pm$  is obtained from the charged Higgs components  $H_u^\pm$ ,  $H_d^\pm$  as  $H^\pm = \cos \beta H_u^\pm + \sin \beta H_d^\pm$ .

Two-loop diagrams involving closed sfermion and chargino loops were studied in [18, 19] and [20], respectively. The leading contributions (photonic Barr-Zee diagrams) can be found in [18, 9] in the context of the MSSM and can be generalized to the NMSSM through a replacement of the corresponding couplings.

For the sfermionic diagrams we use the formula (5) of [18]:

$$\delta a_\mu^{2L\tilde{f}} = \frac{G_F m_\mu^2 \alpha}{4\sqrt{2}\pi^3} \sum_{\tilde{f}} \sum_{i=1}^3 N_c^{\tilde{f}} Q_{\tilde{f}}^2 \frac{\text{Re}[\lambda_\mu^{h_i} \lambda_{\tilde{f}}^{h_i}]}{m_{\tilde{f}}^2} f_{\tilde{f}} \left( \frac{m_{\tilde{f}}^2}{m_{h_i}^2} \right) \quad (2.13)$$

where  $N_c^{\tilde{f}}$  corresponds to the number of colors (3 for squarks, 1 for sleptons),  $Q_{\tilde{f}}$  is the electric charge, and the Higgs/sfermion couplings in the NMSSM are given by:

$$\begin{aligned} \lambda_{\tilde{T}_k}^{h_i} = \frac{2\sqrt{2}M_W}{g_2} & \left[ h_t \{A_t S_{i1} - \lambda (s S_{i2} + v_d S_{i3})\} \text{Re}[X_{k1}^{\tilde{T}*} X_{k2}^{\tilde{T}}] \right. \\ & + \left\{ h_t^2 v_u S_{i1} - \frac{g_1^2}{3} (v_u S_{i1} - v_d S_{i2}) \right\} |X_{k2}^{\tilde{T}}|^2 \\ & + \left. \left\{ h_t^2 v_u S_{i1} - \frac{3g_2^2 - g_1^2}{12} (v_u S_{i1} - v_d S_{i2}) \right\} |X_{k1}^{\tilde{T}}|^2 \right] \quad (2.14) \end{aligned}$$

$$\begin{aligned} \lambda_{\tilde{B}_k}^{h_i} = \frac{2\sqrt{2}M_W}{g_2} & \left[ h_b \{A_b S_{i2} - \lambda (s S_{i1} + v_u S_{i3})\} \text{Re}[X_{k1}^{\tilde{B}*} X_{k2}^{\tilde{B}}] \right. \\ & + \left\{ h_b^2 v_d S_{i2} + \frac{g_1^2}{6} (v_u S_{i1} - v_d S_{i2}) \right\} |X_{k2}^{\tilde{B}}|^2 \\ & + \left. \left\{ h_b^2 v_d S_{i2} + \frac{3g_2^2 + g_1^2}{12} (v_u S_{i1} - v_d S_{i2}) \right\} |X_{k1}^{\tilde{B}}|^2 \right] \quad (2.15) \end{aligned}$$

$$\begin{aligned}
 \lambda_{\tilde{\tau}_k}^{h_i} = & \frac{2\sqrt{2}M_W}{g_2} \left[ h_\tau \{A_\tau S_{i2} - \lambda (sS_{i1} + v_u S_{i3})\} \operatorname{Re}[X_{k1}^{\tilde{\tau}*} X_{k2}^{\tilde{\tau}}] \right. \\
 & + \left\{ h_\tau^2 v_d S_{i2} + \frac{g_1^2}{2} (v_u S_{i1} - v_d S_{i2}) \right\} |X_{k2}^{\tilde{\tau}}|^2 \\
 & \left. + \left\{ h_\tau^2 v_d S_{i2} + \frac{g_2^2 - g_1^2}{4} (v_u S_{i1} - v_d S_{i2}) \right\} |X_{k1}^{\tilde{\tau}}|^2 \right] \quad (2.16)
 \end{aligned}$$

while the muon/Higgs couplings are simply  $\lambda_\mu^{h_i} = \frac{S_{i2}}{\cos\beta}$ .  $v_u$ ,  $v_d$  and  $s$  denote the vacuum expectation values of  $H_u$ ,  $H_d$  and  $S$ , respectively, in the normalisation where  $v_u^2 + v_d^2 = 1/(2\sqrt{2}G_F)$ .  $\lambda$  is the Higgs singlet-doublet Yukawa coupling, and  $A_{t,b,\tau}$  are the soft supersymmetry breaking Higgs-sfermion trilinear couplings for the third generation.

For the closed chargino loop, we employ the formula (63) of [9]:

$$\delta a_\mu^{2L} \chi^\pm = \frac{G_F m_\mu^2 \alpha}{4\sqrt{2}\pi^3} \sum_{k=1}^2 \left[ \sum_{i=1}^3 \frac{\operatorname{Re}[\lambda_\mu^{h_i} \lambda_{\chi_k^\pm}^{h_i}]}{m_{\chi_k^\pm}} f_S \left( \frac{m_{\chi_k^\pm}^2}{m_{h_i}^2} \right) + \sum_{i=1}^2 \frac{\operatorname{Re}[\lambda_\mu^{a_i} \lambda_{\chi_k^\pm}^{a_i}]}{m_{\chi_k^\pm}} f_{\text{PS}} \left( \frac{m_{\chi_k^\pm}^2}{m_{a_i}^2} \right) \right] \quad (2.17)$$

with the chargino/Higgs couplings of the NMSSM:

$$\lambda_{\chi_k^\pm}^{h_i} = \frac{\sqrt{2}M_W}{g_2} [\lambda U_{k2} V_{k2} S_{i3} + g_2 (U_{k1} V_{k2} S_{i1} + U_{k2} V_{k1} S_{i2})] \quad (2.18)$$

$$\lambda_{\chi_k^\pm}^{a_i} = \frac{\sqrt{2}M_W}{g_2} [\lambda U_{k2} V_{k2} P'_{i2} - g_2 (U_{k1} V_{k2} \cos\beta + U_{k2} V_{k1} \sin\beta) P'_{i1}] \quad (2.19)$$

The functions  $f_{\tilde{f}}$ ,  $f_S$  and  $f_{\text{PS}}$  can be found in the appendix.

Among the missing contributions are one-loop diagrams with the exchange of a Higgs boson between the muons, and two-loop diagrams with a closed SM fermion loop involving Higgs bosons (and a photon). Since these can play a particular rôle in the NMSSM, they will be treated separately in the next subsection.

## 2.2 Contributions including pseudoscalars

Higgs effects are usually negligibly small in the SM or the MSSM because of the existing lower bounds on the Higgs masses. The SM one-loop Higgs/muon diagram is, indeed, about four orders of magnitude below the sensitivity of the BNL experiment for a SM Higgs mass above 114 GeV. However, this is not necessarily the case in the NMSSM: while the lightest CP-even Higgs boson mass cannot be far below the LEP bound unless it decouples from the SM sector, the lightest CP-odd boson  $a_1$  can be as light as a few GeV [26, 27]. Bounds from B-physics [30], especially from  $BR(B_s \rightarrow \mu^+ \mu^-)$ , can still be satisfied for low values of  $\tan\beta$  or when the loop-induced  $b - s - a_1$  coupling is suppressed.

In the context of two-Higgs-doublet-models, the impact of light (pseudo-) scalars on  $a_\mu$  was already pointed out in [15, 16, 28]. In the NMSSM, a short analysis of  $a_\mu$  has been performed in [23].

In the following we include contributions from the Higgs sector to  $a_\mu$  up to the two-loop level. One-loop scalar/fermion diagrams have been known for quite a while; see, e.g., [38].



For completeness we detail all the SUSY Higgs contributions (CP-odd, even and charged):

$$\delta a_\mu^{1L \text{ CP even}} = \frac{G_\mu m_\mu^2}{4\sqrt{2}\pi^2} \sum_{i=1}^3 \frac{S_{i2}^2}{\cos^2 \beta} \int_0^1 \frac{x^2(2-x) dx}{x^2 + \left(\frac{m_{h_i}}{m_\mu}\right)^2 (1-x)} \quad (2.20)$$

$$\delta a_\mu^{1L \text{ CP odd}} = -\frac{G_\mu m_\mu^2}{4\sqrt{2}\pi^2} \sum_{i=1}^2 P_{i1}^2 \tan^2 \beta \int_0^1 \frac{x^3 dx}{x^2 + \left(\frac{m_{a_i}}{m_\mu}\right)^2 (1-x)} \quad (2.21)$$

$$\delta a_\mu^{1L \text{ charged}} = \frac{G_\mu m_\mu^2}{4\sqrt{2}\pi^2} \tan^2 \beta \int_0^1 \frac{x(x-1) dx}{x-1 + \left(\frac{m_{H^\pm}}{m_\mu}\right)^2} \quad (2.22)$$

For the two-loop Higgs diagrams involving a closed SM fermion loop (in contrast to closed sfermion/chargino loops considered in the previous subsection), we follow the analysis of [16] and generalize it to the NMSSM:

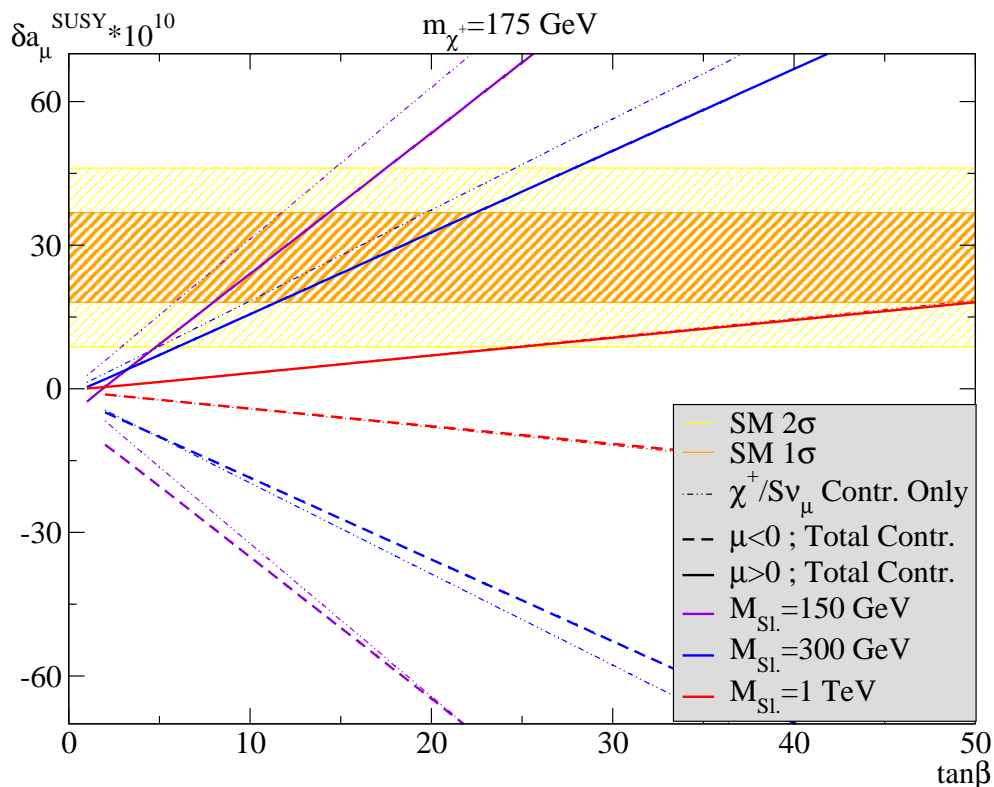
$$\delta a_\mu^{2L \text{ CP even}} = \frac{G_\mu m_\mu^2 \alpha}{4\sqrt{2}\pi^3} \sum_{i=1}^3 \left[ \frac{4}{3} \frac{S_{i1} S_{i2}}{\sin \beta \cos \beta} f_S \left( \frac{m_t^2}{m_{h_i}^2} \right) + \frac{1}{3} \frac{S_{i2}^2}{\cos^2 \beta} f_S \left( \frac{m_b^2}{m_{h_i}^2} \right) + \frac{S_{i2}^2}{\cos^2 \beta} f_S \left( \frac{m_\tau^2}{m_{h_i}^2} \right) \right] \quad (2.23)$$

$$\delta a_\mu^{2L \text{ CP odd}} = \frac{G_\mu m_\mu^2 \alpha}{4\sqrt{2}\pi^3} \sum_{i=1}^2 P_{i1}^2 \left[ \frac{4}{3} f_{\text{PS}} \left( \frac{m_t^2}{m_{a_i}^2} \right) + \tan^2 \beta \left\{ \frac{1}{3} f_{\text{PS}} \left( \frac{m_b^2}{m_{a_i}^2} \right) + f_{\text{PS}} \left( \frac{m_\tau^2}{m_{a_i}^2} \right) \right\} \right] \quad (2.24)$$

where the functions  $f_S$ ,  $f_{\text{PS}}$  are defined in the appendix.

As noticed in [28], one-loop and two-loop light Higgs contributions are of opposite signs and interfere, therefore, destructively. In the case of a CP-odd scalar, the one-loop contribution is negative and worsens the discrepancy (1.9) correspondingly. However, for a light CP-odd Higgs heavier than  $\sim 3$  GeV, the positive two-loop contribution is numerically more important. The sum of both contributions is maximal around  $m_{a_1} \sim 6$  GeV, though fairly constant in the range 4 – 10 GeV (see figure 5 below). Both one- and two-loop contributions are proportional to the product of two muon Yukawa couplings, which leads to an enhancement quadratic in  $\tan \beta$ . They are also proportional to the square of the mixing of the light pseudoscalar to the doublet sector ( $\sim (P'_{11} \sin \beta)^2$ , if we consider the dominant  $H_d$  component only). In spite of the appearance of  $P'_{11}$ , this coupling can be large enough to allow the light pseudoscalar contribution to reach the experimental  $2\sigma$  range of  $a_\mu$  by itself, provided that  $\tan \beta \gtrsim 30$ . Hence, this contribution can alleviate the upper bound on slepton masses, which can be derived under the assumption that the chargino contribution (2.1) explains the deviation (1.9).

Finally, we estimate the theoretical uncertainty following the analysis of [9], allowing for a 2% error on the one-loop contributions and a relative error of 30% for the two-loop results. Except for the light pseudoscalar contributions, which we include in the error



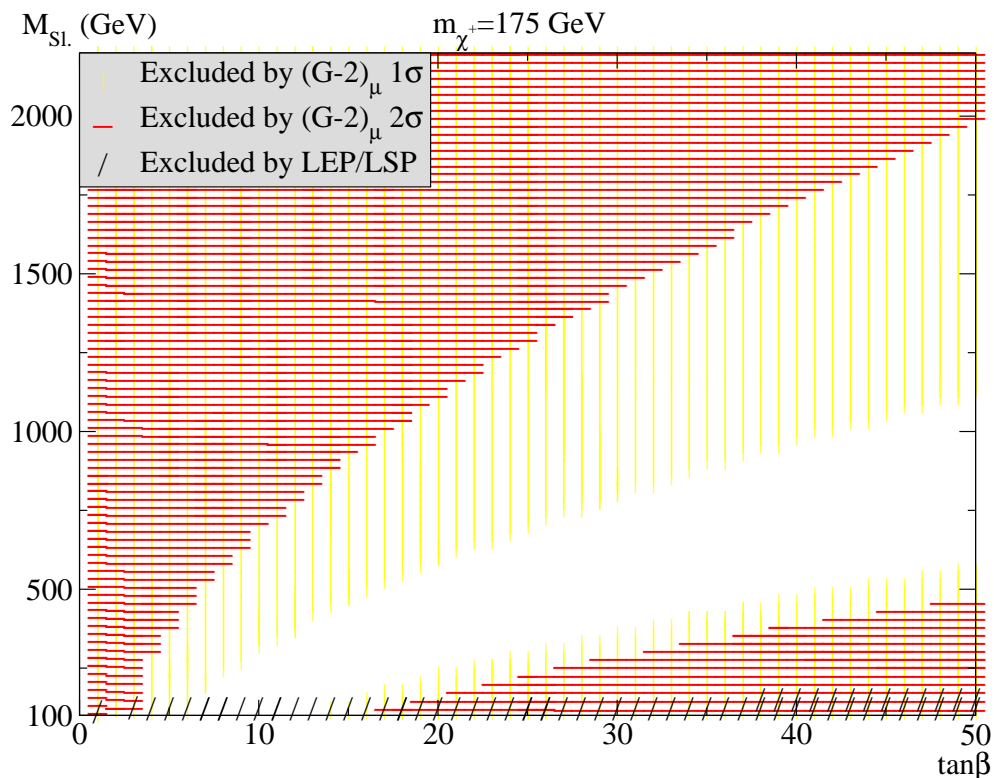
**Figure 1:** The SUSY contribution to  $a_\mu$  as a function of  $\tan\beta$  for various slepton (sneutrino/smuon) masses.

computation described above, we do not expect large sources of uncertainties different from the MSSM. Therefore, we use the same additional constant terms as [9] for the evaluation of the error.

### 3. Results

In this section we discuss a few phenomenological aspects of the supersymmetric contributions to  $a_\mu$  discussed above. First we consider the  $\tan\beta$  and slepton mass dependences in NMSSM scenarios without a light CP-odd scalar; as expected, these results are essentially the same as in the MSSM. Then we examine the case of a light pseudoscalar and show how relevant the Higgs contribution may become. Finally we study the cNMSSM (with universal soft terms at the GUT scale), and conclude with a short summary.

In figure 1, we plot the supersymmetric contribution  $a_\mu^{\text{SUSY}}$  as a function of  $\tan\beta$  for various slepton (sneutrino/smuon) masses. Here we chose, for simplicity, universal soft slepton masses  $M_{\text{SL}}$  at low energy. The gaugino soft terms are assumed to be hierarchical ( $M_3 = 3M_2 = 6M_1 = 900$  GeV) and  $\mu_{\text{eff}}$  is chosen such that the lighter chargino  $\chi^+$  has a mass of 175 GeV. The experimentally allowed  $1\sigma$  and  $2\sigma$  regions are indicated as an orange and a yellow band, respectively. The full violet, blue or red curves (corresponding to  $\mu_{\text{eff}} > 0$ ) and dashed violet, blue or red curves (corresponding to  $\mu_{\text{eff}} < 0$ ) include all SUSY

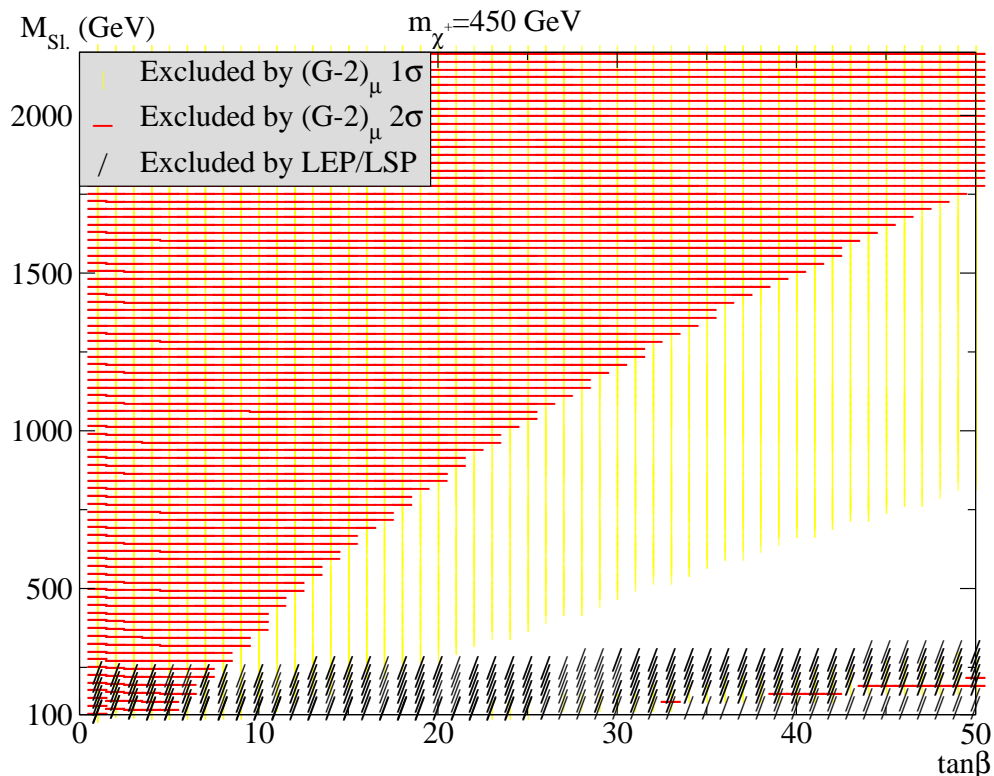


**Figure 2:** Exclusion plot in the  $\tan\beta$ /slepton mass plane for a light chargino of 175 GeV.

contributions. Next to them we plot as dot-dashed lines the one-loop chargino/sneutrino contribution separately (corrected by large QED logarithms; for sneutrino/smuon masses of 1 TeV they differ hardly from the full SUSY contributions). As we mentioned in the previous section, the chargino/sneutrino contribution is proportional to the Yukawa coupling of the muon and thus to  $\tan\beta$ . Since this contribution obviously dominates the total SUSY contribution (the small difference is mainly due to the neutralino diagram), the total SUSY contribution is also roughly proportional to  $\tan\beta$ .

The chargino contribution to  $a_\mu$  carries the same sign as the SUSY parameter  $\mu$  ( $\mu_{\text{eff}}$  in the NMSSM). Hence, the case  $\mu < 0$  (dashed curves in figure 1) is disfavoured by the sign of the difference of the BNL result and the SM (1.9). When  $\mu$  is positive (full curves in figure 1), the SUSY contribution to  $a_\mu$  is able to account for the  $3\sigma$  deviation. When sleptons are heavy (the red curve), however, large values of  $\tan\beta$  are necessary. On the other hand, for light sleptons (violet curve) the chargino contribution can be large enough even for moderate values of  $\tan\beta$ , or even too important when  $\tan\beta$  is large.

In figure 2 we present an exclusion plot in the  $\tan\beta$ /slepton mass plane for a light chargino of 175 GeV. In the red and yellow hatched domains the muon magnetic moment differs from the BNL result at the  $2\sigma$  and  $1\sigma$  level, respectively. (In the excluded dark hatched region a slepton would be the LSP, or violate constraints from its non-observation at LEP.) Two different domains are excluded by  $a_\mu$ :



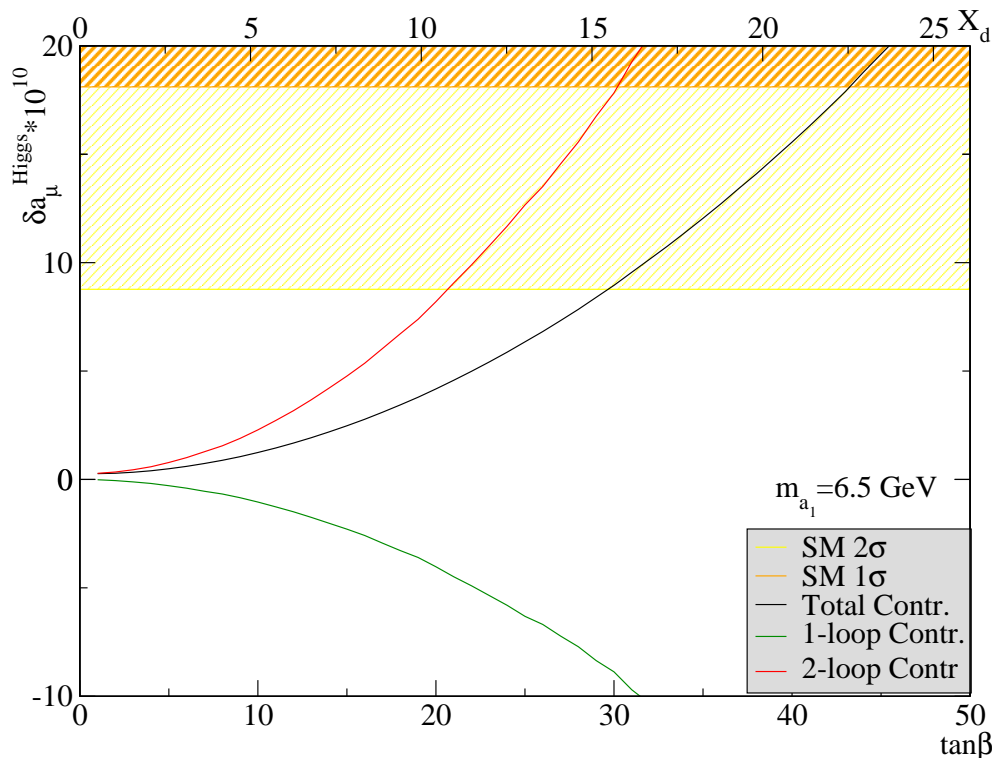
**Figure 3:** Exclusion plot in the  $\tan\beta$ /slepton mass plane for a heavier chargino of 450 GeV.

- In the bottom right-hand corner of figure 2,  $\tan\beta$  is large and sleptons are light. In this case, the chargino/sneutrino contribution is strongly enhanced and becomes too large.
- When sleptons are heavy and  $\tan\beta$  is low (top and left-hand side of figure 2), the chargino contribution to  $a_{\mu}$  is suppressed, and SUSY cannot explain the BNL result.

In order to see the impact of the chargino mass, we consider the case of a heavier chargino of 450 GeV in figure 3. Here we chose  $M_3 = 3M_2 = 6M_1 = 1.5 \text{ TeV}$ . One sees that light sleptons and large values of  $\tan\beta$  are required in order to account for the BNL result.

These results confirm that the chargino/sneutrino contribution can explain the  $3\sigma$  deviation between  $a_{\mu}^{\text{SM}}$  and  $a_{\mu}^{\text{EXP}}$  in the NMSSM as well as in the MSSM, provided that the supersymmetric particles are sufficiently light or  $\tan\beta$  is large.

Next we consider the NMSSM with a light pseudoscalar  $a_1$  in the spectrum. From the formulae (2.21) and (2.24) one finds that its contribution to  $a_{\mu}$  depends essentially on the pseudoscalar mass  $m_{a_1}$  and its coupling to (down) fermions  $X_d = P'_{11} \tan\beta$ , where  $P'_{11}$  describes the mixing of  $a_1$  with the Higgs-doublet sector. Subsequently we chose parameters in the Higgs sector such that  $P'_{11}$  remains approximately constant  $\sim 0.52$ ; then  $X_d$  is entirely determined by  $\tan\beta$ .

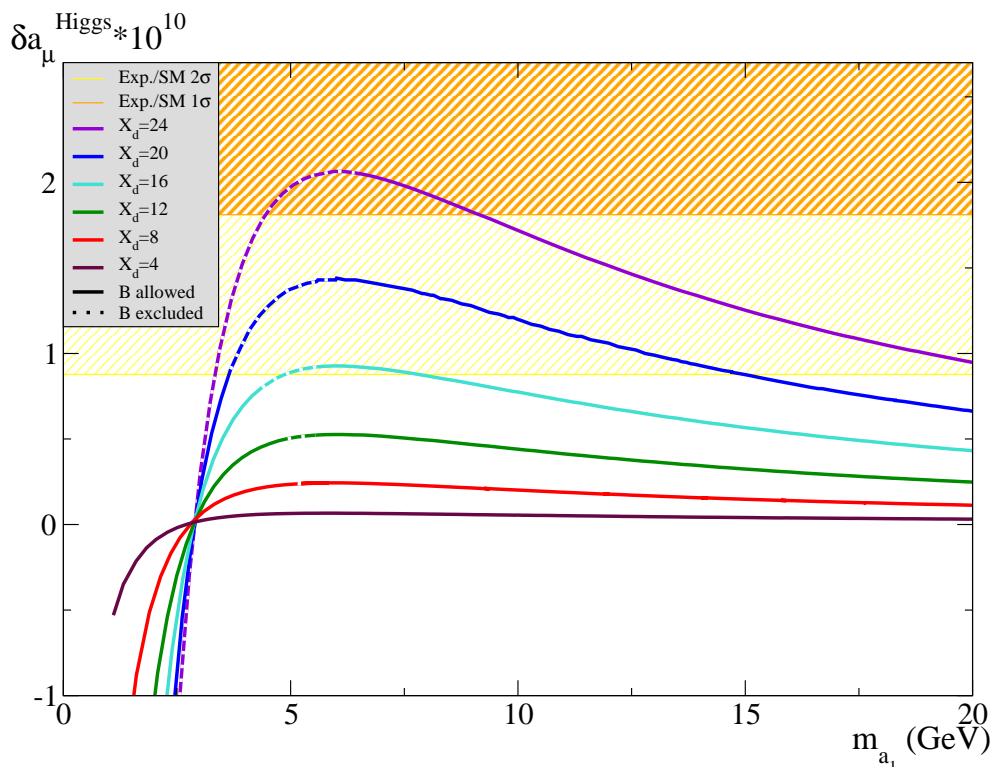


**Figure 4:** Higgs contribution to  $a_\mu$  as a function of  $\tan\beta$  or  $X_d$  for  $m_{a_1} = 6.5$  GeV.

In figure 4, we plot the contribution to  $a_\mu$  which originates from the NMSSM Higgs sector only against  $\tan\beta$  (lower axis) or  $X_d$  (upper axis) for  $m_{a_1} = 6.5$  GeV; then the contributions in section 2.2 are dominated by the light pseudoscalar. We indicate separately the negative one-loop contribution (green curve), and the positive two-loop contribution (red curve). For  $m_{a_1} = 6.5$  GeV the two-loop contribution dominates, so that the total result (black curve) has the same sign as the desired contribution (1.9). The contribution behaves as  $\tan^2\beta$  (or  $X_d^2$ ) and we find that the Higgs contribution alone can reduce the discrepancy (1.9) below the  $2\sigma$  ( $1\sigma$ ) level for  $X_d \gtrsim 15$  (22.5).

In figure 5 we show the contribution to  $a_\mu$  from the NMSSM Higgs sector as a function of  $m_{a_1}$  for various values of  $X_d$ . As we mentioned in the previous section, the negative one-loop contribution dominates below  $m_{a_1} \sim 3$  GeV, while the two-loop diagram dominates for larger masses. Since both contributions decrease when the pseudoscalar becomes heavy, the total Higgs contribution becomes maximal around  $\sim 6 - 7$  GeV.

Constraints from B physics (essentially from  $\bar{B}_s \rightarrow \mu^+\mu^-$  and  $\Delta M_s$ ) depend mainly on the loop-induced  $b-s-a_1$  coupling [30], and are particularly strong for  $m_{a_1} \sim M_{B_{d,s}} \sim 5$  GeV. For fixed  $X_d$  (and sfermion masses and trilinear couplings at 1 TeV,  $\lambda = 0.3$ ,  $\mu_{\text{eff}} = 200$  GeV, and  $M_3 = 3M_2 = 6M_1 = 1.2$  TeV) we varied the remaining parameters  $\tan\beta$ ,  $\kappa$  and  $A_\kappa$  in the Higgs sector of the NMSSM and obtained regions near  $m_{a_1} \sim 5$  GeV, which are *always* excluded — these regions are indicated as dashed parts of the curves in figure 5. For larger  $X_d$  and corresponding larger values of  $\tan\beta$ , the forbidden regions are larger. For



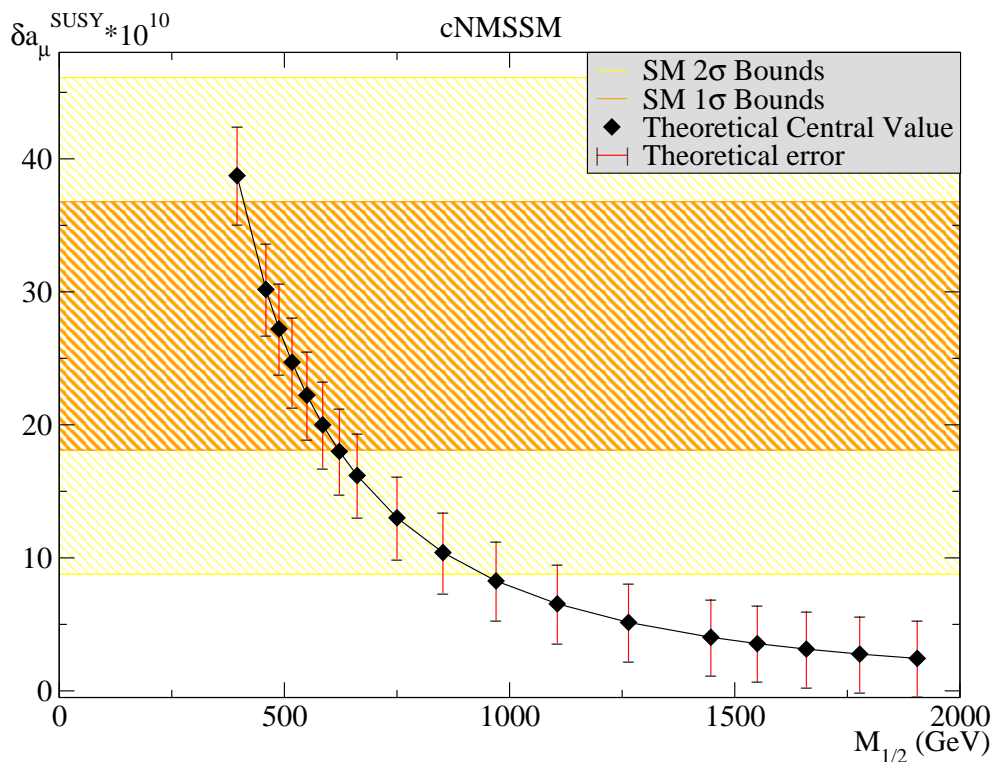
**Figure 5:** Higgs contribution to  $a_\mu$  as a function of the pseudoscalar mass.

$X_d = 24$ , the forbidden region extends up to  $m_{a_1} \sim 6.5$  GeV. The region  $m_{a_1} \gtrsim 6.5$  GeV, where the total Higgs contribution can be relatively large, can be consistent with B physics constraints, however.

The previous results concern certain regions of the relatively large parameter space of the general (low energy) NMSSM. Next we consider the impact of  $a_\mu$  on the cNMSSM with universal scalar and gaugino masses ( $m_0$  and  $M_{1/2}$ ) as well as universal trilinear couplings ( $A_0$ ) at the GUT scale. Such a model is motivated by flavour-blind, supergravity-induced SUSY breaking scenarios.

A recent study of the cNMSSM parameter space spanned by  $\lambda$ ,  $m_0$ ,  $M_{1/2}$  and  $A_0$  [39] showed that LEP constraints on Higgs scalars and, notably, a dark matter relic density in agreement with WMAP constraints require  $\lambda \ll 1$ ,  $m_0 \ll M_{1/2}$  and that  $A_0$  is determined in terms of  $M_{1/2}$ . Consequently, the complete Higgs and sparticle spectrum depends essentially only on  $M_{1/2}$ , which can *a priori* vary between 400 GeV and 2–3 TeV (where all other constraints on sparticle masses and from B-physics are satisfied; the lower limit on  $M_{1/2}$  originates simultaneously from the lower experimental bound on stau masses and LEP constraints on Higgs scalars).

Using the most recent version of NMSSMTools [35], we have computed the spectrum and the supersymmetric contributions to  $a_\mu$  in the cNMSSM as a function of  $M_{1/2}$ , with the result shown in figure 6.



**Figure 6:**  $\delta a_\mu$  as a function of  $M_{1/2}$  in the cNMSSM

We see that the constraint from  $a_\mu$  confines the allowed range of  $M_{1/2}$  to  $M_{1/2} \lesssim 1$  TeV at the  $2\sigma$  level, and to  $400 \text{ GeV} < M_{1/2} \lesssim 700 \text{ GeV}$  (where the sparticle spectrum is not too heavy) at the  $1\sigma$  level. In fact, the present experimental value could be matched to arbitrarily high precision, and a more precise measurement of  $a_\mu$  could determine  $M_{1/2}$  completely.

In the cNMSSM, the leading SUSY contributions to  $a_\mu$  originate from the MSSM-like one-loop diagrams. Besides the chargino/sneutrino loop, the bino/smuon contribution is also quite significant. On the other hand, contributions from the Higgs sector are always negligible in the cNMSSM, since the lightest pseudoscalar is not very light.

To summarize, the deviation (1.9) of the measured value of  $a_\mu$  from the SM can be explained in the NMSSM as in the MSSM since – in the absence of a light CP-odd Higgs scalar – the corresponding contributions are practically the same. In the presence of a light CP-odd Higgs scalar the NMSSM specific contributions to  $a_\mu$  are not negligible in general (depending on  $X_d$  and  $m_{a_1}$ , see figures 4 and 5) and, for  $m_{a_1} > 3 \text{ GeV}$ , can allow for a heavier sparticle spectrum. The mSUGRA-like cNMSSM is consistent with constraints from  $a_\mu$ , which can help to confine the remaining free parameter  $M_{1/2}$ .

## A. Loop functions

The functions appearing in the one-loop contributions (2.1), (2.2) are given in [11]:

$$\begin{aligned}
 F_1^N(x) &= \frac{2}{(1-x)^4} [1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x] \\
 F_2^N(x) &= \frac{3}{(1-x)^3} [1 - x^2 + 2x \ln x] \\
 F_1^C(x) &= \frac{2}{(1-x)^4} [2 + 3x - 6x^2 + x^3 + 6x \ln x] \\
 F_2^C(x) &= -\frac{3}{2(1-x)^3} [3 - 4x + x^2 + 2 \ln x],
 \end{aligned}$$

The functions appearing in the two-loop contributions (2.13), (2.17, 2.23, 2.24) are as in [9]:

$$\begin{aligned}
 f_{\tilde{f}}(z) &= -\frac{z}{2} \int_0^1 \frac{x(1-x)}{x(1-x)-z} \ln \frac{x(1-x)}{z} dx = \frac{z}{2} [2 + \ln z - f_{\text{PS}}(z)] \\
 f_{\text{PS}}(z) &= z \int_0^1 \frac{1}{x(1-x)-z} \ln \frac{x(1-x)}{z} dx \\
 &= \frac{2z}{\sqrt{1-4z}} \left[ \text{Li}_2 \left( 1 - \frac{1 - \sqrt{1-4z}}{2z} \right) - \text{Li}_2 \left( 1 - \frac{1 + \sqrt{1-4z}}{2z} \right) \right] \\
 f_S(z) &= -z \int_0^1 \frac{1 - 2x(1-x)}{x(1-x)-z} \ln \frac{x(1-x)}{z} dx \\
 &= (2z - 1) f_{\text{PS}}(z) - 2z (\ln z + 2)
 \end{aligned}$$

## References

- [1] MUON G-2 collaboration, G.W. Bennett et al., *Final report of the muon E821 anomalous magnetic moment measurement at BNL*, *Phys. Rev.* **D 73** (2006) 072003 [[hep-ex/0602035](#)].
- [2] F. Jegerlehner, *Precision measurements of sigma(hadronic) for alpha\_eff(E) at ILC energies and (g-2)(mu)*, *Nucl. Phys.* **162** (Proc. Suppl.) (2006) 22 [[hep-ph/0608329](#)].
- [3] K. Hagiwara, A.D. Martin, D. Nomura and T. Teubner, *Improved predictions for g-2 of the muon and alpha\_QED(M\_Z^2)*, *Phys. Lett.* **B 649** (2007) 173 [[hep-ph/0611102](#)].
- [4] M. Davier, *The hadronic contribution to (g-2)(mu)*, *Nucl. Phys.* **169** (Proc. Suppl.) (2007) 288 [[hep-ph/0701163](#)].
- [5] F. Jegerlehner, *Essentials of the muon g-2*, *Acta Phys. Polon.* **B38** (2007) 3021 [[hep-ph/0703125](#)].
- [6] Z. Zhang, *Muon g-2: a mini review*, [arXiv:0801.4905](#).
- [7] R. Alemany, M. Davier and A. Hocker, *Improved determination of the hadronic contribution to the muon (g-2) and to alpha(M(Z)^2) using new data from hadronic tau decays*, *Eur. Phys. J.* **C 2** (1998) 123 [[hep-ph/9703220](#)].
- [8] M. Davier, S. Eidelman, A. Hocker and Z. Zhang, *Updated estimate of the muon magnetic moment using revised results from e+e- annihilation*, *Eur. Phys. J.* **C 31** (2003) 503 [[hep-ph/0308213](#)].



- [9] D. Stöckinger, *The muon magnetic moment and supersymmetry*, *J. Phys.* **G 34** (2007) R45 [[hep-ph/0609168](#)].
- [10] P. Fayet, *Supersymmetry, particle physics and gravitation*, in *Unification of the Fundamental Particles Interactions*, S. Ferrara, J. Ellis and P. van Nieuwenhuizen eds., Plenum Press, New York U.S.A (1980), pg. 587;  
 J.A. Grifols and A. Mendez, *Constraints on supersymmetric particle masses from  $(g - 2)\mu$* , *Phys. Rev.* **D 26** (1982) 1809;  
 J.R. Ellis, J.S. Hagelin and D.V. Nanopoulos, *Spin 0 leptons and the anomalous magnetic moment of the muon*, *Phys. Lett.* **B 116** (1982) 283;  
 R. Barbieri and L. Maiani, *The muon anomalous magnetic moment in broken supersymmetric theories*, *Phys. Lett.* **B 117** (1982) 203;  
 D.A. Kosower, L.M. Krauss and N. Sakai, *Low-energy supergravity and the anomalous magnetic moment of the muon*, *Phys. Lett.* **B 133** (1983) 305;  
 T.C. Yuan, R. Arnowitt, A.H. Chamseddine and P. Nath, *Supersymmetric electroweak effects on  $G - 2(\mu)$* , *Z. Physik* **C 26** (1984) 407;  
 J.C. Romao, A. Barroso, M.C. Bento and G.C. Branco, *Flavor violation in supersymmetric theories*, *Nucl. Phys.* **B 250** (1985) 295;  
 I. Vendramin, *Constraints on supersymmetric parameters from muon magnetic moment*, *Nuovo Cim.* **A101** (1989) 731;  
 S.A. Abel, W.N. Cottingham and I.B. Whittingham, *The muon magnetic moment in flipped SU(5)*, *Phys. Lett.* **B 259** (1991) 307;  
 U. Chattopadhyay and P. Nath, *Probing supergravity grand unification in the Brookhaven  $g - 2$  experiment*, *Phys. Rev.* **D 53** (1996) 1648 [[hep-ph/9507386](#)];  
 T. Moroi, *The muon anomalous magnetic dipole moment in the minimal supersymmetric standard model*, *Phys. Rev.* **D 53** (1996) 6565 [*Erratum ibid.* **56** (1997) 4424] [[hep-ph/9512396](#)].
- [11] S.P. Martin and J.D. Wells, *Muon anomalous magnetic dipole moment in supersymmetric theories*, *Phys. Rev.* **D 64** (2001) 035003 [[hep-ph/0103067](#)].
- [12] M. Byrne, C. Kolda and J.E. Lennon, *Updated implications of the muon anomalous magnetic moment for supersymmetry*, *Phys. Rev.* **D 67** (2003) 075004 [[hep-ph/0208067](#)].
- [13] S.P. Martin and J.D. Wells, *Super-conservative interpretation of muon  $g - 2$  results applied to supersymmetry*, *Phys. Rev.* **D 67** (2003) 015002 [[hep-ph/0209309](#)].
- [14] G. Degrandi and G.F. Giudice, *QED logarithms in the electroweak corrections to the muon anomalous magnetic moment*, *Phys. Rev.* **D 58** (1998) 053007 [[hep-ph/9803384](#)].
- [15] D. Chang, W.-F. Chang, C.-H. Chou and W.-Y. Keung, *Large two-loop contributions to  $g - 2$  from a generic pseudoscalar boson*, *Phys. Rev.* **D 63** (2001) 091301 [[hep-ph/0009292](#)].
- [16] K.-m. Cheung, C.-H. Chou and O.C.W. Kong, *Muon anomalous magnetic moment, two-Higgs-doublet model and supersymmetry*, *Phys. Rev.* **D 64** (2001) 111301 [[hep-ph/0103183](#)].
- [17] C.-H. Chen and C.Q. Geng, *The muon anomalous magnetic moment from a generic charged Higgs with SUSY*, *Phys. Lett.* **B 511** (2001) 77 [[hep-ph/0104151](#)].
- [18] A. Arhrib and S. Baek, *Two-loop Barr-Zee type contributions to  $(g - 2)(\mu)$  in the MSSM*, *Phys. Rev.* **D 65** (2002) 075002 [[hep-ph/0104225](#)].

- [19] S. Heinemeyer, D. Stöckinger and G. Weiglein, *Two-loop SUSY corrections to the anomalous magnetic moment of the muon*, *Nucl. Phys. B* **690** (2004) 62 [[hep-ph/0312264](#)].
- [20] S. Heinemeyer, D. Stöckinger and G. Weiglein, *Electroweak and supersymmetric two-loop corrections to  $(g-2)(\mu)$* , *Nucl. Phys. B* **699** (2004) 103 [[hep-ph/0405255](#)].
- [21] T.-F. Feng, X.-Q. Li, L. Lin, J. Maalampi and H.-S. Song, *The two-loop supersymmetric corrections to lepton anomalous magnetic and electric dipole moments*, *Phys. Rev. D* **73** (2006) 116001 [[hep-ph/0604171](#)].
- [22] T.-F. Feng, L. Sun and X.-Y. Yang, *Electroweak and supersymmetric two-loop corrections to lepton anomalous magnetic and electric dipole moments*, *Nucl. Phys. B* **800** (2008) 221 [[arXiv:0805.1122](#)].
- [23] J.F. Gunion, D. Hooper and B. McElrath, *Light neutralino dark matter in the NMSSM*, *Phys. Rev. D* **73** (2006) 015011 [[hep-ph/0509024](#)].
- [24] V. Barger, C. Kao, P. Langacker and H.-S. Lee, *Muon anomalous magnetic moment in a supersymmetric  $U(1)'$  model*, *Phys. Lett. B* **614** (2005) 67 [[hep-ph/0412136](#)].
- [25] H.P. Nilles, M. Srednicki and D. Wyler, *Weak interaction breakdown induced by supergravity*, *Phys. Lett. B* **120** (1983) 346;  
 J.M. Frere, D.R.T. Jones and S. Raby, *Fermion masses and induction of the weak scale by supergravity*, *Nucl. Phys. B* **222** (1983) 11;  
 J.P. Derendinger and C.A. Savoy, *Quantum effects and  $SU(2) \times U(1)$  breaking in supergravity gauge theories*, *Nucl. Phys. B* **237** (1984) 307;  
 J.R. Ellis, J.F. Gunion, H.E. Haber, L. Roszkowski and F. Zwirner, *Higgs bosons in a nonminimal supersymmetric model*, *Phys. Rev. D* **39** (1989) 844;  
 M. Drees, *Supersymmetric models with extended Higgs sector*, *Int. J. Mod. Phys. A* **4** (1989) 3635.
- [26] B.A. Dobrescu, G.L. Landsberg and K.T. Matchev, *Higgs boson decays to CP-odd scalars at the Tevatron and beyond*, *Phys. Rev. D* **63** (2001) 075003 [[hep-ph/0005308](#)];  
 B.A. Dobrescu and K.T. Matchev, *Light axion within the next-to-minimal supersymmetric standard model*, *JHEP* **09** (2000) 031 [[hep-ph/0008192](#)].
- [27] U. Ellwanger, J.F. Gunion, C. Hugonie and S. Moretti, *Towards a no-lose theorem for NMSSM Higgs discovery at the LHC*, [hep-ph/0305109](#);  
 LHC/LC STUDY GROUP collaboration, G. Weiglein et al., *Physics interplay of the LHC and the ILC*, *Phys. Rept.* **426** (2006) 47 [[hep-ph/0410364](#)];  
 U. Ellwanger, J.F. Gunion, C. Hugonie and S. Moretti, *NMSSM Higgs discovery at the LHC*, [hep-ph/0401228](#);  
 HIGGS WORKING GROUP collaboration, K.A. Assamagan et al., *The Higgs working group: summary report 2003*, [hep-ph/0406152](#);  
 R. Dermisek and J.F. Gunion, *Escaping the large fine tuning and little hierarchy problems in the next to minimal supersymmetric model and  $h \rightarrow aa$  decays*, *Phys. Rev. Lett.* **95** (2005) 041801 [[hep-ph/0502105](#)]; *Consistency of LEP event excesses with an  $h \rightarrow aa$  decay scenario and low-fine-tuning NMSSM models*, *Phys. Rev. D* **73** (2006) 111701 [[hep-ph/0510322](#)]; *The NMSSM close to the R-symmetry limit and naturalness in  $h \rightarrow aa$  decays for  $m(a) < 2m(b)$* , *Phys. Rev. D* **75** (2007) 075019 [[hep-ph/0611142](#)];  
 U. Ellwanger, J.F. Gunion and C. Hugonie, *Difficult scenarios for NMSSM Higgs discovery at the LHC*, *JHEP* **07** (2005) 041 [[hep-ph/0503203](#)];

- S. Chang, P.J. Fox and N. Weiner, *Naturalness and Higgs decays in the MSSM with a singlet*, *JHEP* **08** (2006) 068 [[hep-ph/0511250](#)];
- P.W. Graham, A. Pierce and J.G. Wacker, *Four taus at the Tevatron*, [hep-ph/0605162](#);
- S. Moretti, S. Munir and P. Poulose, *Another step towards a no-lose theorem for NMSSM Higgs discovery at the LHC*, *Phys. Lett.* **B 644** (2007) 241 [[hep-ph/0608233](#)];
- S. Chang, P.J. Fox and N. Weiner, *Visible cascade Higgs decays to four photons at hadron colliders*, *Phys. Rev. Lett.* **98** (2007) 111802 [[hep-ph/0608310](#)];
- T. Stelzer, S. Wiesenfeldt and S. Willenbrock, *Higgs at the Tevatron in extended supersymmetric models*, *Phys. Rev.* **D 75** (2007) 077701 [[hep-ph/0611242](#)];
- U. Aglietti et al., *Tevatron-for-LHC report: Higgs*, [hep-ph/0612172](#);
- E. Fullana and M.-A. Sanchis-Lozano, *Hunting a light CP-odd non-standard Higgs boson through its tauonic decay at a (super) B factory*, *Phys. Lett.* **B 653** (2007) 67 [[hep-ph/0702190](#)];
- K. Cheung, J. Song and Q.-S. Yan, *Role of  $h \rightarrow \eta\eta$  in Intermediate-Mass Higgs Boson Searches at the Large Hadron Collider*, *Phys. Rev. Lett.* **99** (2007) 031801 [[hep-ph/0703149](#)];
- M.-A. Sanchis-Lozano, *A light non-standard Higgs boson: to be or not to be at a (super) B factory?*, [arXiv:0709.3647](#);
- M. Carena, T. Han, G.-Y. Huang and C.E.M. Wagner, *Higgs Signal for  $h \rightarrow aa$  at hadron colliders*, *JHEP* **04** (2008) 092 [[arXiv:0712.2466](#)];
- J.R. Forshaw, J.F. Gunion, L. Hodgkinson, A. Papaefstathiou and A.D. Pilkington, *Reinstating the 'no-lose' theorem for NMSSM Higgs discovery at the LHC*, *JHEP* **04** (2008) 090 [[arXiv:0712.3510](#)];
- Z. Heng, R.J. Oakes, W. Wang, Z. Xiong and J.M. Yang, *B meson dileptonic decays in NMSSM with a light CP-odd Higgs boson*, *Phys. Rev.* **D 77** (2008) 095012 [[arXiv:0801.1169](#)];
- A. Djouadi et al., *Benchmark scenarios for the NMSSM*, *JHEP* **07** (2008) 002 [[arXiv:0801.4321](#)];
- X.-G. He, J. Tandean and G. Valencia, *Rare decays with a light CP-odd Higgs boson in the NMSSM*, *JHEP* **06** (2008) 002 [[arXiv:0803.4330](#)].
- [28] M. Krawczyk, *Precision muon  $g-2$  results and light Higgs bosons in the 2HDM(II)*, *Acta Phys. Polon.* **B33** (2002) 2621 [[hep-ph/0208076](#)].
- [29] ALEPH, L3 and OPAL collaboration, S. Schael et al., *Search for neutral MSSM Higgs bosons at LEP*, *Eur. Phys. J.* **C 47** (2006) 547 [[hep-ex/0602042](#)].
- [30] F. Domingo and U. Ellwanger, *Updated constraints from B physics on the MSSM and the NMSSM*, *JHEP* **12** (2007) 090 [[arXiv:0710.3714](#)].
- [31] S. Laporta and E. Remiddi, *The analytical value of the electron  $(g-2)$  at order  $\alpha^3$  in QED*, *Phys. Lett.* **B 379** (1996) 283 [[hep-ph/9602417](#)];
- T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, *Revised value of the eighth-order electron  $g-2$* , *Phys. Rev. Lett.* **99** (2007) 110406 [[arXiv:0706.3496](#)];
- T. Kinoshita and M. Nio, *Improved  $\alpha^4$  term of the muon anomalous magnetic moment*, *Phys. Rev.* **D 70** (2004) 113001 [[hep-ph/0402206](#)].
- [32] T. Kinoshita and M. Nio, *The tenth-order QED contribution to the lepton  $g-2$ : evaluation of dominant  $\alpha^5$  terms of muon  $g-2$* , *Phys. Rev.* **D 73** (2006) 053007 [[hep-ph/0512330](#)].
- [33] J. Bijnens and J. Prades, *The hadronic light-by-light contribution to the muon anomalous magnetic moment: where do we stand?*, *Mod. Phys. Lett.* **A 22** (2007) 767 [[hep-ph/0702170](#)].

- [34] A. Czarnecki, W.J. Marciano and A. Vainshtein, *Refinements in electroweak contributions to the muon anomalous magnetic moment*, *Phys. Rev. D* **67** (2003) 073006 [*Erratum ibid.* **73** (2006) 119901] [[hep-ph/0212229](#)].
- [35] U. Ellwanger, J.F. Gunion and C. Hugonie, *NMHDECAY: a Fortran code for the Higgs masses, couplings and decay widths in the NMSSM*, *JHEP* **02** (2005) 066 [[hep-ph/0406215](#)];  
U. Ellwanger and C. Hugonie, *NMSPEC: a Fortran code for the sparticle and Higgs masses in the NMSSM with GUT scale boundary conditions*, *Comput. Phys. Commun.* **177** (2007) 399 [[hep-ph/0612134](#)];  
<http://www.th.u-psud.fr/NMHDECAY/nmssmtools.html>.
- [36] P. Skands et al., *SUSY Les Houches accord: interfacing SUSY spectrum calculators, decay packages and event generators*, *JHEP* **07** (2004) 036 [[hep-ph/0311123](#)].
- [37] B. Allanach et al., *SUSY Les Houches accord 2*, [arXiv:0801.0045](#).
- [38] J.P. Leveille, *The second order weak correction to  $(g - 2)$  of the muon in arbitrary gauge models*, *Nucl. Phys. B* **137** (1978) 63.
- [39] A. Djouadi, U. Ellwanger and A.M. Teixeira, *The constrained next-to-minimal supersymmetric standard model*, [arXiv:0803.0253](#).